Trigonometry Identities

Degrees and Radians

If θ is an angle in degrees and α is the same angle in radians:

$$\theta = \alpha \times \frac{180}{\pi} \qquad \alpha = \theta \times \frac{\pi}{180}$$

Odd or Even Identities

$$\sin(-\theta) = -\sin(\theta) \qquad \csc(-\theta) = -\csc(\theta)$$
$$\cos(-\theta) = \cos(\theta) \qquad \sec(-\theta) = \sec(\theta)$$
$$\tan(-\theta) = -\tan(\theta) \qquad \cot(-\theta) = -\cot(\theta)$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Complementary Angles (Degrees)

$$\tan(90^\circ - \theta) = \cot\theta$$
$$\cot(90^\circ - \theta) = \tan\theta$$
$$\sin(90^\circ - \theta) = \cos\theta$$
$$\csc(90^\circ - \theta) = \sec\theta$$
$$\cos(90^\circ - \theta) = \sin\theta$$
$$\sec(90^\circ - \theta) = \csc\theta$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

Complementary Angles (Radians)

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$
$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$$
$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta \qquad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$$

Periodic Identities

If n is an integer

$$\tan(\theta + \pi n) = \tan\theta \qquad \cot(\theta + \pi n) = \cot\theta$$
$$\sin(\theta + 2\pi n) = \sin\theta \qquad \csc(\theta + 2\pi n) = \csc\theta$$
$$\cos(\theta + 2\pi n) = \cos\theta \qquad \sec(\theta + 2\pi n) = \sec\theta$$

Double Angle Identities

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$= \frac{2\tan\theta}{1 + \tan^2\theta}$$
$$= (\cos\theta + \sin\theta)^2 - 1$$
$$= 1 - (\cos\theta - \sin\theta)^2$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$
$$= \frac{1 - \tan^2\theta}{1 + \tan^2\theta}$$

Triple Angles

$$\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$$
$$\cos(3\theta) = 4\cos^3\theta - 3\cos\theta$$
$$\tan(3\theta) = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

Sum and Difference Identities

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$
$$\cot(\alpha \pm \beta) = \frac{\cot\alpha\cot\beta \mp 1}{\cot\beta \pm \cot\alpha}$$
$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

Half Angle Identities

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos\theta}{2}}$$

or

$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 + \cos\theta}{2}}$$

or

$$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$$

$$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}}$$
$$= \frac{1 - \cos\theta}{\sin\theta}$$
$$= \frac{\sin\theta}{1 + \cos\theta}$$

or

$$\tan^2\theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

Product to Sum Identities

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$
$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$
$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha - \beta) + \sin(\alpha + \beta)]$$
$$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

Sum and Difference to Product Identities

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$$
$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$
$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$$
$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$

Further Tangent Identities

$$\tan(45^\circ - \theta) = \frac{1 - \tan\theta}{1 + \tan\theta}$$
$$\tan(45^\circ + \theta) = \frac{1 + \tan\theta}{1 - \tan\theta}$$
$$\tan\left(\frac{\pi}{4} - \theta\right) = \frac{1 - \tan\theta}{1 + \tan\theta}$$
$$\tan\left(\frac{\pi}{4} + \theta\right) = \frac{1 + \tan\theta}{1 - \tan\theta}$$

Trigonometric Functions in terms of the other Ratios

$$\tan\theta = \pm\frac{\sqrt{1 - \cos^2\theta}}{\cos\theta} = \pm\frac{\sin\theta}{\sqrt{1 - \sin^2\theta}} = \frac{1}{\cot\theta} = \pm\sqrt{\sec^2\theta - 1} = \pm\frac{1}{\sqrt{\csc^2\theta - 1}}$$

$$\sin\theta = \pm\frac{\tan\theta}{\sqrt{1 + \tan^2\theta}} = \pm\sqrt{1 - \cos^2\theta} = \pm\frac{1}{\sqrt{1 + \cot^2\theta}} = \pm\frac{\sqrt{\sec^2\theta - 1}}{\sec\theta} = \frac{1}{\csc\theta}$$

$$\cos\theta = \pm\frac{1}{\sqrt{1 + \tan^2\theta}} = \pm\sqrt{1 - \sin^2\theta} = \pm\frac{\cot\theta}{\sqrt{1 + \cot^2\theta}} = \frac{1}{\sec\theta} = \pm\frac{\sqrt{\csc^2\theta - 1}}{\csc\theta}$$