Calculus

Definite Integral Definition

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x$$

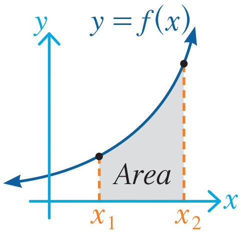
where f is continuous on $[a, b]$ and $F' = f$

Fundamental Theorem of Calculus

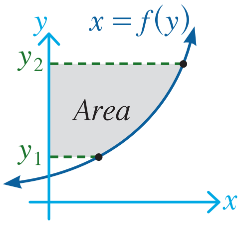
$$\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$$

where f is continuous on $[a, b]$ and $F' = f$

Areas under the function

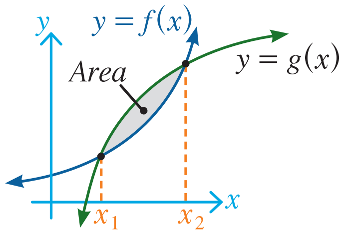


$$A = \int_{x_1}^{x_2} f(x)dx$$
$$= [F(x)]_{x_1}^{x_2}$$
$$= F(x_2) - F(x_1)$$



$$A = \int_{y_1}^{y_2} f(y)dy$$
$$= [F(y)]_{y_1}^{y_2}$$
$$= F(y_2) - F(y_1)$$

Area between two functions



$$A = \int_{x_1}^{x_2} g(x) - f(x)dx$$
$$= [G(x) - F(x)]_{x_1}^{x_2}$$

$f(x)$ and $g(x)$ are both continuous on $[x_1, x_2]$, $g(x) > f(x)$ on $[x_1, x_2]$ and $F'(x) = f(x)$, $G'(x) = g(x)$

Standard Integrals

$$\int k\, dx = kx + C$$

$$\int x^n\, dx = \frac{1}{n+1}x^{n+1} + C$$

$$\int x^{-1}\, dx = \int \frac{1}{x} dx = \ln|x| + C$$

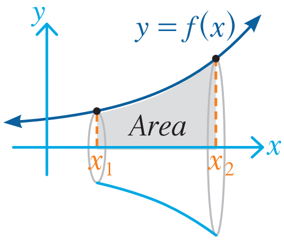
$$\int e^x\, dx = e^x + C$$

$$\int \ln x\, dx = x\ln(x) - x + C$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a}\ln|ax+b| + C$$

$$\int \frac{1}{a^2+x^2} dx = \log\left(x + \sqrt{x^2+a^2}\right) + C$$

Volume of a Solid of Revolution



$$\text{Volume} = \pi \int_{x_1}^{x_2} [radius]^2 dx = \pi \int_{x_1}^{x_2} [f(x)]^2 dx$$

where $f(x)$ is continuous on $[x_1, x_2]$

Intergration Properties

$$\int_a^b cf(x)dx = c\int_a^b f(x)dx$$

$$\int_a^a f(x)dx = 0$$

$$\int_a^b -f(x)dx = -\int_a^b f(x)dx$$

$$\int_a^b f(x) \pm g(x)dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

$$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$$

Inequalities

If $f(x) < g(x)$ for all $x \in [a, b]$, then:

$$\int_a^b f(x)dx \le \int_a^b g(x)dx$$

$$\left(\int_0^b f(x)g(x)dx\right)^2 \le \left(\int_0^b [g(x)]^2 dx\right)\left(\int_0^b [g(x)]^2 dx\right)$$

Trigonometric Integrals

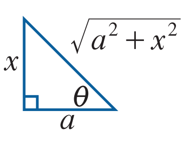
$\int \cos x\, dx = \sin x + C$	$\int \csc^2 x\, dx = -\cot x + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sec x \tan x\, dx = \sec x + C$
$\int \sec^2 x\, dx = \tan x + C$	$\int \csc x \cot x\, dx = -\csc x + C$
$\int \tan x\, dx = \ln\|\sec x\| + C$	$\int \sec x\, dx = \ln\|\sec x + \tan x\| + C$
$\int \cot x\, dx = \ln\|\sin x\| + C$	$\int \csc x\, dx = \ln\|\csc x - \cot x\| + C$

Intergration by Parts

$$\int u\, dv = uv - \int v\, du \quad \text{where} \quad v = \int dv$$

$$\int f(x)g'(x)\, dx = f(x)g(x) - \int f'(x)g(x)\, dx$$

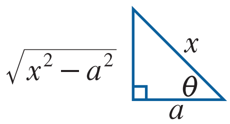
Trigonometric Substitutions



$x = a\tan\theta$

$dx = a\sec^2\theta\ d\theta$

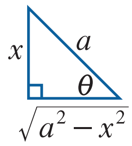
$\sqrt{a^2 + a^2\tan^2\theta}$
$= a\sec\theta$



$x = a\sec\theta$

$dx = a\sec\theta\tan\theta\ d\theta$

$\sqrt{a^2\sec^2\theta - a^2}$
$= a\tan\theta$



$x = a\sin\theta$

$dx = a\cos\theta\ d\theta$

$\sqrt{a^2 - a^2\sin^2\theta}$
$= a\cos\theta$

$$\int \frac{1}{a^2+x^2}\, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\, dx = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int -\frac{1}{\sqrt{a^2-x^2}}\, dx = \cos^{-1}\left(\frac{x}{a}\right) + C$$

Intergration by Substitution

$$\int_a^b f(g(x))\ g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$$

where $u = g(x)$ and $du = g'(x)dx$