Algebra

Operations, Inequalities and Exponent Laws

Arithmetic Rules in Algebra

Commutative: $a+b=b+a$, $a\times b=b\times a$

Associative: $(a+b)+c=a+(b+c)$, $a\times(b\times c)=(a\times b)\times c$

Distributive: $a(b+c)=ab+ac$, $a(b-c)=ab-ac$

Arithmetic Operations in Algebra

$ab+ac=a(b+c)$

$ab-ac=a(b-c)$

$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$

$\frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}$

$\frac{ad+ac}{a}=d+c,\ a\neq 0$

$\frac{ad-ac}{a}=d-c,\ a\neq 0$

$\frac{a+b}{c}=\frac{a}{c}+\frac{b}{c},\ c\neq 0$

$\frac{a-b}{c}=\frac{a}{c}-\frac{b}{c},\ c\neq 0$

$\frac{a}{b}\div c=\frac{\left(\frac{a}{b}\right)}{c}=\frac{a}{b}\times\frac{1}{c}=\frac{a}{bc}$

$a\div\frac{b}{c}=\frac{a}{\left(\frac{b}{c}\right)}=a\times\frac{c}{b}=\frac{ac}{b}$

$\frac{a-b}{c-d}=\frac{-(b-a)}{-(d-c)}=\frac{b-a}{d-c}$

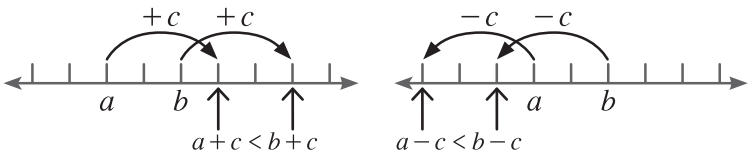
$a\times\frac{b}{c}=\frac{ab}{c}$

$\frac{a}{b}\times\frac{c}{d}=\frac{a\times c}{b\times d}=\frac{ac}{bd}$

$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)}=\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc}$

Inequalities

If $a<b$, then $a+c<b+c$ and $a-c<b-c$

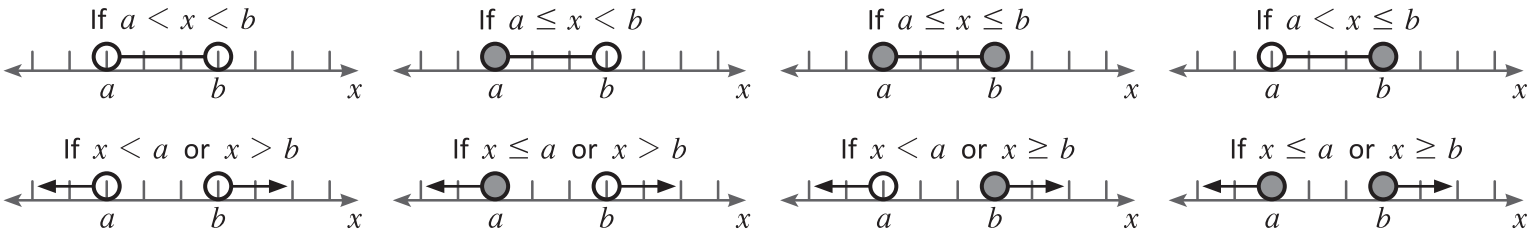


If $a<b$, and $c>0$ then:

- $ac<bc$
- $\frac{a}{c}<\frac{b}{c}$

If $a<b$, and $c<0$ then:

- $ac>bc$
- $\frac{a}{c}>\frac{b}{c}$



Absolute Value

$|a|=\begin{cases} a, & \text{if } a\geq 0 \\ -a, & \text{if } a<0 \end{cases}$

$|a|=|-a|$

$|a|\geq 0$

$|a+b|\leq|a|+|b|$

$|ab|=|a|\times|b|$

$\left|\frac{a}{b}\right|\geq\frac{|a|}{|b|}=|a\div b|=|a|\div|b|$

Exponent Laws and their Variations

$a^n=a\times a\times a\ldots n$ times

$a^1=a$

$a^0=1$, where $a\in R,\ a\neq 0$

$a^m\times a^n=a^m a^n=a^{m+n}$

If $a^m=a^n$ and $a\neq\pm 1$, and $a\neq 0$, then $m=n$

If $a^m=b^m$ and $m\neq 0$, then $a=\pm b$

$\frac{a^m}{a^n}=a^m\div a^n=a^{m-n}$ if $m>n$

$=1$ if $m=n$

$=\frac{1}{a^{n-m}}$ if $m<n;\ a\in R,\ a\neq 0$

$(a^m)^n=a^{m\times n}=a^{mn}$

$(ab)^m=a^m\times b^m=a^m b^m$

$\left(\frac{a}{b}\right)^m=\frac{a^m}{b^m}$

$a^{-m}=\frac{1}{a^m},a\neq 0$

$\frac{1}{a^{-m}}=1\div\frac{1}{a^m}=1\times\frac{a^m}{1}=a^m,a\neq 0$

$\left(\frac{a}{b}\right)^{-m}=\frac{a^{-m}}{b^{-m}}=\frac{\left(\frac{1}{a^m}\right)}{\left(\frac{1}{b^m}\right)}=\frac{1}{a^m}\div\frac{1}{b^m}=\frac{1}{a^m}\times\frac{b^m}{1}=\frac{b^m}{a^m}=\left(\frac{b}{a}\right)^m$

$a^{\frac{m}{n}}=(a^{\frac{1}{n}})^m=(a^m)^{\frac{1}{n}}=\sqrt[n]{a^m}=(\sqrt[n]{a})^m$

$a^{-\left(\frac{m}{n}\right)}=\frac{1}{a^{\frac{m}{n}}}=\frac{1}{(a^{\frac{1}{n}})^m}=\frac{1}{(a^m)^{\frac{1}{n}}}=\frac{1}{\sqrt[n]{a^m}}=\frac{1}{(\sqrt[n]{a})^m}$

$\left(\frac{a}{b}\right)^{\frac{m}{n}}=\frac{a^{\frac{m}{n}}}{b^{\frac{m}{n}}}=\frac{\sqrt[n]{a^m}}{\sqrt[n]{b^m}}=\frac{(\sqrt[n]{a})^m}{(\sqrt[n]{b})^m}=\left(\frac{\sqrt[n]{a}}{\sqrt[n]{b}}\right)^m=\left(\frac{a^{\frac{1}{n}}}{b^{\frac{1}{n}}}\right)^m$